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## **Assessment Framework for Grade 10 Math**

### **November 2004**

- **Designed to Provide Teachers with Grade 10 an assessment framework aligned to Montana content standards.**
- **Grade eight topics may appear on the grade 10 framework as topics through grade 10 can be prior to grade 10.**
- **Released items and the key to standards may provide more concrete examples.**

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## **Mathematics Content Standard One**

**Students engage in the mathematical processes of problem solving and reasoning, estimation, communications, connections and applications, and using appropriate technology.**

### **Montana “By Graduation” Benchmark 1:**

**Recognize and formulate problems from situations within and outside mathematics and apply solution strategies to those problems.**

**For grade ten, this means:**

- Recognize and apply algebra techniques and geometric principles to problem solving.
- Understand how mathematical ideas interconnect and build on one another.

### **Montana “By Graduation” Benchmark 2:**

**Select, apply, and evaluate appropriate estimation strategies throughout the problem solving process.**

**For grade ten, this means:**

- Select and use appropriate methods for computing with real numbers in problem-solving situations from among mental arithmetic, estimation, paper-and-pencil, calculator, and computer methods; and determine whether the results are reasonable.
- Determine when estimation is an appropriate method to solve a problem and describe what error might result from estimation.

### **Montana “By Graduation” Benchmark 3:**

**Formulate definitions, make and justify inferences, express generalizations, and communicate mathematical ideas and relationships.**

**For grade ten, this means:**

- Formulate definitions of algebraic concepts (e.g., slope and intercept).
- Translate problems into equations or inequalities.
- Demonstrate basic mathematical logic (e.g., conditionals and converses).
- Formulate definitions of geometric concepts in more advanced terms.

## **Mathematics Content Standard One, continued**

### **Montana “By Graduation” Benchmark 4:**

**Apply and translate among different representations of the same problem situation or of the same mathematical concept. Model connections between problem situations that arise in disciplines other than mathematics.**

**For grade ten, this means:**

- Use ratios, proportions, and percent in problem solving situations.
- Convert from one set of units to another (e.g., feet per minute to miles per hour).
- Apply direct variation to problem solving situations.
- Recognize and use connections among mathematical ideas.
- Translate problems into equations or inequalities.
- Apply inverse variation to problem-solving situations.

### **Montana “By Graduation” Benchmark 5:**

**Select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, computer, and data collection devices.**

**For grade ten, this means:**

- Select and use appropriate technology to solve basic algebra and geometry problems.
- Judge the reasonableness of the results of symbolic manipulations, including those carried out by technology.

## **Mathematics Content Standard Two**

**Students demonstrate understanding of and an ability to use numbers and operations.**

### **Montana “By Graduation” Benchmark 1:**

**Use and understand the real number system, its operations, notations, and the various subsystems.**

**For grade ten, this means:**

- Demonstrate meaning for real numbers, absolute value, and scientific notation using physical materials and technology in problem-solving situations.
- Compare and order sets of real numbers.
- Recognize and use equivalent representations of real numbers in a variety of forms including scientific notation, radicals, and other irrational numbers such as  $\pi$ .
- Identify and use the properties of operations on real numbers, including the associative, commutative, distributive, and closure properties, the existence of the identity and inverse elements for addition and multiplication, the existence of the  $n^{\text{th}}$  roots of positive real numbers for any positive integer, and the inverse relationship between taking the  $n^{\text{th}}$  root of and the  $n^{\text{th}}$  power of a real number.

### **Montana “By Graduation” Benchmark 2:**

**Use definitions and basic operations of the complex number systems.**

**For grade ten, this means:**

- Understand complex numbers as solutions to quadratic equations that do not have real solutions.

### **Mathematics Content Standard Three**

**Students use algebraic concepts, processes, and language to model and solve a variety of real-world and mathematical problems.**

#### **Montana “By Graduation” Benchmark 1:**

**Use algebra to represent patterns of change.**

**For grade ten, this means:**

- Demonstrate an understanding of the relationship between various representations of a line (e.g., tables, graphs, equations).
- Determine a line’s slope and intercepts from its graph or from a linear equation.
- Find a linear equation from a graph or a geometric description of the line (e.g., by using point-slope or slope-intercept formulas).
- Explain the significance of a positive, negative, zero, or undefined slope.
- Generalize patterns using explicitly defined or recursively defined functions.
- Understand the difference between slope and steepness (i.e., steepness = absolute value of slope).

#### **Montana “By Graduation” Benchmark 2:**

**Use basic operations with algebraic expressions.**

**For grade ten, this means:**

- Add, subtract, and multiply polynomials. Divide polynomials by monomials.
- Factor second degree polynomials.
- Identify and use the properties of operations on real numbers (including associative, commutative, distributive and closure).
- Evaluate functions.

#### **Montana “By Graduation” Benchmark 3:**

**Solve algebraic equations and inequalities: linear, quadratic, exponential, logarithmic, and power.**

**For grade ten, this means:**

- Write and solve linear equations and inequalities.
- Solve absolute value linear equations.
- Find solutions to quadratic equations (with real roots) by factoring, or using the quadratic formula.
- Solve simple power equations (for example:  $x^5=7$ ).
- Write equivalent forms of equations and inequalities (e.g., solve  $I=PRT$  for  $T$ ).

## **Mathematics Content Standard Three, continued**

### **Montana “By Graduation” Benchmark 4:**

**Solve systems of algebraic equations and inequalities, including use of matrices.**

**For grade ten, this means:**

- Write and solve systems of linear equations and inequalities.
- Apply algebraic and graphical methods to find the solution (examples: mixture, rate, age problems).

### **Montana “By Graduation” Benchmark 5:**

**Use algebraic models to solve mathematical and real-world problems.**

**For grade ten, this means:**

- Identify essential qualitative relationships in a situation and determine the class or classes of functions (e.g., linear or quadratic) that might model the relationship.
- Solve problems that can be modeled using linear or quadratic functions.
- Express the perimeter, area, and volume relationships of geometric figures symbolically.

## **Mathematics Content Standard Four**

**Students demonstrate understanding of shape and an ability to use geometry**

### **Montana “By Graduation” Benchmark 1:**

**Construct, interpret, and draw three-dimensional objects.**

**For grade ten, this means:**

- Identify three-dimensional objects (e.g. cylinders, cones).
- Visualize three-dimensional objects from different perspectives (e.g., front, top, or side view) and analyze their cross sections.

### **Montana “By Graduation” Benchmark 2:**

**Classify figures in terms of congruence and similarity and apply these relationships.**

**For grade ten, this means:**

- Apply congruence and similarity correspondences and properties of figures to find missing parts of geometric figures and provide logical justification.

## **Mathematics Content Standard Four, continued**

### **Montana “By Graduation” Benchmark 3:**

**Translate between synthetic and coordinate representations.**

**For grade ten, this means:**

- Specify location using coordinates geometry and other representational systems (e.g. representing a polynomial’s vertices on the coordinate plane).
- Transfer dimensioned shapes to a coordinate plane.

### **Montana “By Graduation” Benchmark 4:**

**Deduce properties of figures using transformations, coordinates, and vectors in problem solving.**

**For grade ten, this means:**

- Investigate conjectures and solve problems involving two-dimensional objects represented with Cartesian coordinates (e.g., calculate midpoints of segments, slopes of lines and segments, and distances between two points, and apply the results to the solutions of problems).
- Draw the results and interpret transformations on figures in the coordinate plane (e.g. translations, reflections, rotations, dilations, and the results of successive transformations).
- Recognize the connections between transformations and congruence, similarity, line symmetry, and rotational symmetry.
- Apply geometric relationships such as parallelism and perpendicularity, numerical relationships on a triangle, relationships between triangles, and properties of quadrilaterals and regular polygons.

### **Mathematics Content Standard Four, continued**

### **Montana “By Graduation” Benchmark 5:**

**Apply trigonometric ratios (sine, cosine, and tangent) to problem situations involving triangles.**

**For grade ten, this means:**

- Use sine, cosine, and tangent ratios to find lengths and angle measures.

## **Mathematics Content Standard Five**

**Students demonstrate understanding of measurable attributes and an ability to use measurement process.**

### **Montana “By Graduation” Benchmark 1:**

**Apply concepts of indirect measurements (e.g. using similar triangles to calculate a distance.**

**For grade ten, this means:**

- Measuring quantities indirectly using techniques of algebra, geometry, or trigonometry (e.g., use formulas for perimeter, area, surface area, and volume for geometric figures).
- Use appropriate measurements to solve problems indirectly (for example, find the height of a flagpole using similar triangles or right triangle trigonometry).
- Use the rate of change to model real-world problems algebraically or graphically.
- Describe how changing the measure of one attribute of a geometric figure affects the other measurements.

### **Montana “By Graduation” Benchmark 2:**

**Use dimensional analysis to check reasonableness of procedures.**

**For grade ten, this means:**

- Given commonly used multi-dimensional figures, determine what units and measurement need to be taken.
- Use unit analysis to check measurement computations.

### **Montana “By Graduation” Benchmark 3:**

**Investigate systems of derived measures (e.g., km/sec, g/cm<sup>3</sup>).**

**For grade ten, this means:**

- Use measurement to solve real-world problems involving rate of change (for example, distance traveled using rate and time).
- Make decisions about units and scales that are appropriate for problem situations involving measurement.



## **Mathematics Content Standard Five, continued**

### **Montana “By Graduation” Benchmark 4:**

**Apply the appropriate concepts of estimates in measurement, error in measurement, tolerance, and precision.**

**For grade ten, this means:**

- Select and use appropriate tools and techniques to measure quantities in order to achieve specified degrees of precision, accuracy, and error of measurements.
- Determine the degree of accuracy of a measurement (for example, by understanding and using significant digits).
- Determine when estimation is an appropriate method to solve a problem and describe what error might result from estimation.

## **Mathematics Content Standard Six**

**The students demonstrate understanding of an ability to use data analysis, probability, and statistics.**

### **Montana “By Graduation” Benchmark 1:**

**Use curve fitting to make predictions from data.**

**For grade ten, this means:**

- Approximate a line of best fit (trend line) given a set of data (e.g., scatterplot). Use technology when appropriate.
- Make predictions from data (e.g., area under a curve).

### **Montana “By Graduation” Benchmark 2:**

**Apply measures of central tendency and demonstrate understanding of the concepts of variability and correlation.**

**For grade ten, this means:**

- Select, create, and interpret appropriate graphical representations for a set of data including scatterplots, tables, stem-and-leaf plots, box-and-whisker plots, circle graphs, line graphs, line plots, and histograms/bar graphs.
- Use appropriate statistics (e.g., mean, median, mode, range) to communicate information about the data.
- Compare different sets of data.
- Recognize how changes in data can affect the measures of central tendency (e.g., replacing the maximum with a higher value and determining which statistics change).

## **Mathematics Content Standard Six, continued**

### **Montana “By Graduation” Benchmark 3:**

**Select an appropriate sampling method for a given statistical analysis.**

**For grade ten, this means:**

- Know the characteristics of well-designed studies, including the roles of randomization in surveys and experiments, sample selection and size, and validity.
- Demonstrate how the responses to a survey can be affected by the way the questions are phrased and/or by the reader’s bias.

### **Montana “By Graduation” Benchmark 4:**

**Use experimental probability, theoretical probability, and simulation methods to represent and solve problems, including expected values.**

**For grade ten, this means:**

- Distinguish between experimental and theoretical probability and use each appropriately.
- Differentiate between independent and dependent events to calculate the probability in real-world situations.
- Calculate the probability of a compound event (e.g., A and B, A or B)
- Use area models to determine probability (for example, the probability of hitting the bull’s eye region in a target).
- Solve real-world problems with informal use of combinations and permutations as counting techniques (e.g., determining the number of possible meals at a restaurant featuring a given number of side dishes).

### **Montana “By Graduation” Benchmark 5:**

**Design a statistical experiment to study a problem and communicate the outcomes.**

**For grade ten, this means:**

- Select, create, and interpret appropriate graphical representations for a set of data including scatterplots, tables, histograms, stem-and-leaf plots, box-and-whisker plots, circle graphs, line graphs, and line plots
- Check a graph, table, or summary for misleading characteristics.

### **Montana “By Graduation” Benchmark 6:**

**Describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.**

**For grade ten, this means:**

- Know that the mean lies at the center of a bell-shaped curve

## **Mathematics Content Standard Seven**

**Students demonstrate understanding of and an ability to use patterns, relations, and functions.**

### **Montana “By Graduation” Benchmark 1:**

**Describe functions and their inverses using graphical, numerical, physical, algebraic, and verbal mathematics models or representations.**

**For grade ten, this means:**

- Recognize and create multiple representations (written explanations, tables, equations, and graphs) of the same linear real-life situations.
- Convert from one representation to another.
- Interpret a graphical representation of a real-world situation.
- Understand relations and functions.

### **Montana “By Graduation” Benchmark 2:**

**Analyze the graphs of the families of polynomial, rational, power, exponential, logarithmic, and periodic functions.**

**For grade ten, this means:**

- Analyze and compare the classes of functions (e.g. linear, quadratics, exponential).
- Using a graph, identify the maximum and minimum value within a given domain.
- Determine the slope and intercepts of a linear equation represented by a graph or an equation, and interpret the meaning of those values relative to the context of the problem.

### **Montana “By Graduation” Benchmark 3:**

**Analyze the effects of parameter changes on the graphs of functions and relations, including translations.**

**For grade ten, this means:**

- Demonstrate horizontal and vertical translations on graphs of functions.
- Determine how a change in one variable affects the other variable in a linear, quadratic, or exponential function.

## **Mathematics Content Standard Seven, continued**

### **Montana “By Graduation” Benchmark 4:**

**Model real-world phenomena with a variety of functions.**

**For grade ten, this means:**

- Model real world phenomena involving linear, quadratic, and exponential relationships using multiple representations of rules that can take the form of a recursive process, a function, an equation, or an inequality.

### **Montana “By Graduation” Benchmark 5:**

**Use graphing for parametric equations, three-dimensional equations, and recursive relations.**

**For grade ten, this means:**

- Represent rules for patterns, both those that can be explicitly defined and those that need to be defined recursively, using words, algebraic expressions, or equations.